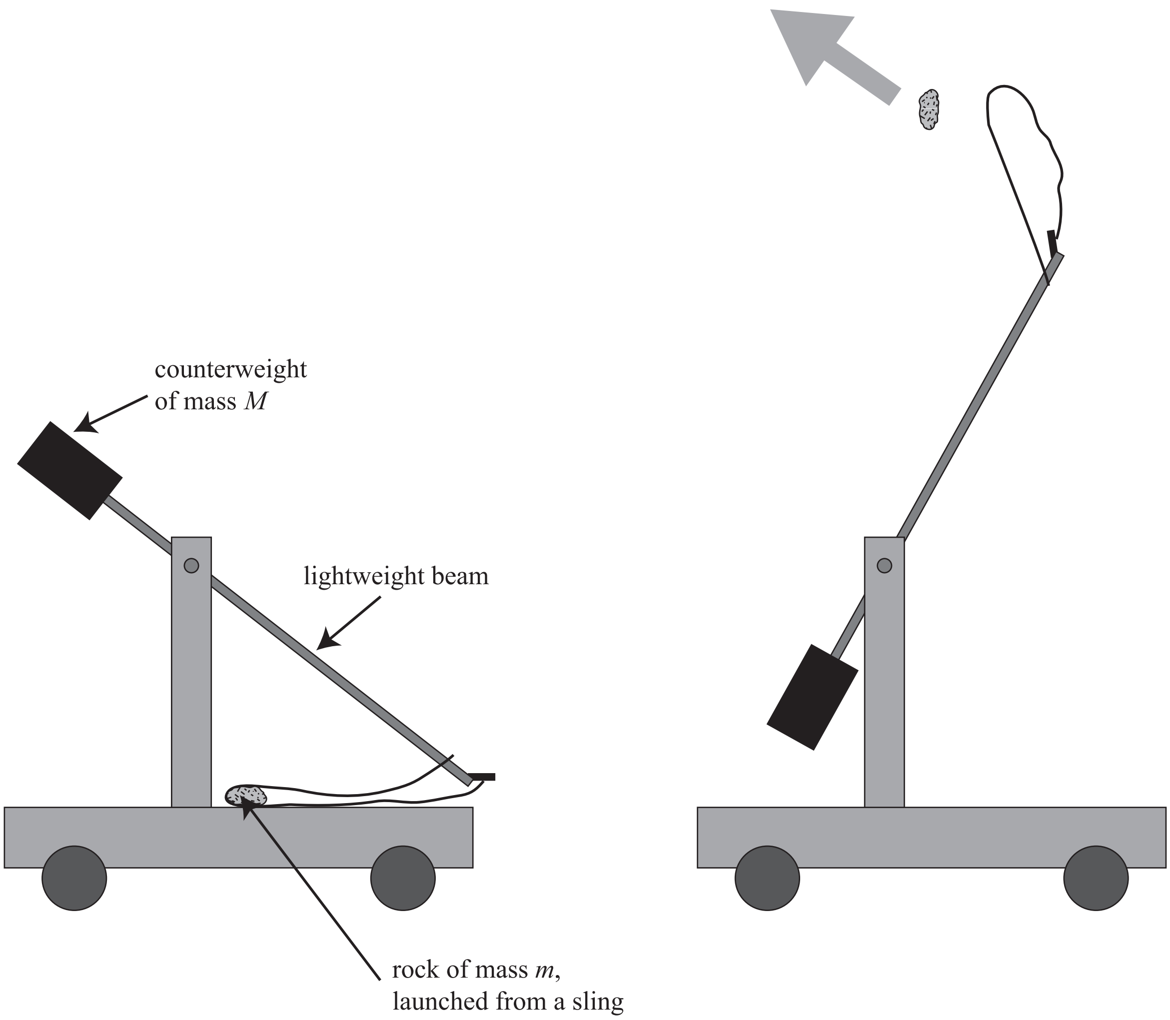
QUESTION THREE: THE TREBUCHET (8 marks)

Assessor's use only

Acceleration due to gravity = 9.81 m s^{-2}

The trebuchet is a medieval weapon for hurling rocks at fortifications.



(a) State the energy changes that take place when the machine fires the rock.

Assessor's use only

(b) Assuming that the rock is released from ground level, show that the theoretical maximum range is:

$R = 2\frac{M}{m}h$, where M = mass of counterweight
m = mass of rock
h = height counterweight falls

(c) The maximum range can be increased by mounting the trebuchet on wheels (rather than fixing it to the ground).

Explain.

(d) If a trebuchet with a maximum range of 100 m on Earth were taken to the Moon (where the gravitational field strength is one sixth of that on the surface of the Earth), what would be its range?

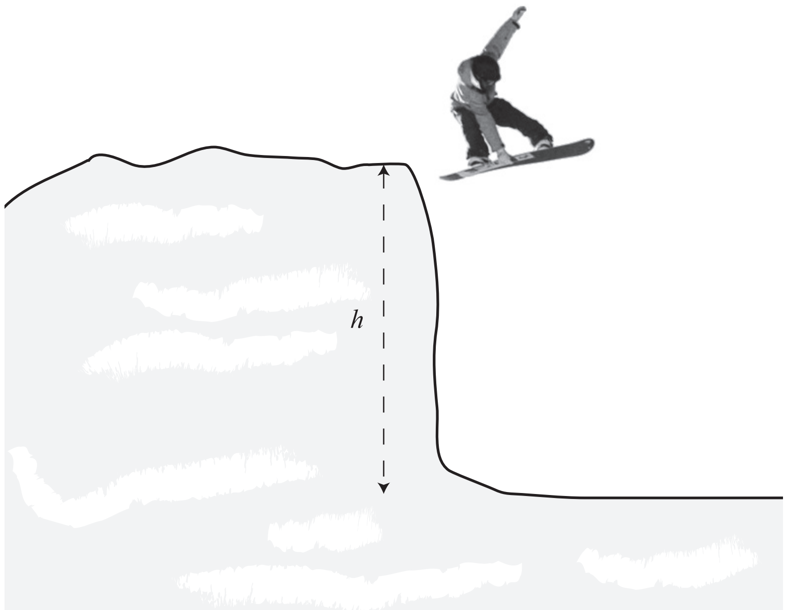
Using physical principles, explain your answer.

(8)

Assessor's use only

QUESTION SIX: SNOWBOARDING (8 marks)

A snowboarder of mass m rides over an icy ledge onto a horizontal surface below.
The snowboard leaves the ledge at 0 m s^{-1} in the vertical direction and at only a very small horizontal velocity.



(a) Assuming that the centre of mass drops by a distance b (through the bending of the knees) on impact, show that the average reaction force acting on the snowboarder is

$$F_R = mg\left(1 + \frac{h}{b}\right)$$

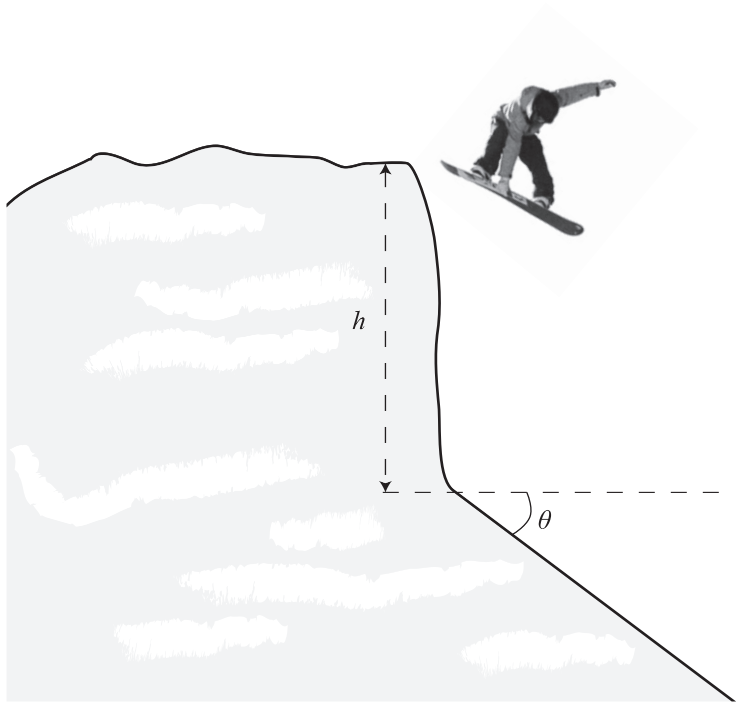
Assessor's use only

(b) By using a height, h, of 3 m and a reasonable estimate of b, calculate the size of the average reaction force experienced by the snowboarder. Comment on how the actual force might differ from the average force.

(c) Show that the time to come to a stop is given by $t = b\sqrt{\frac{2}{gh}}$ and discuss the effect of landing on soft snow (a sample calculation is required).

Question Six (d) is on the following page.

(d) Having survived his first fall, the intrepid snowboarder makes sure his next fall is onto a surface sloping at angle θ to the horizontal, as shown.



It can be shown that $F_R = mg\left(1+\frac{h}{b}\right)\cos\theta$.

Explain, using physical principles, the effect of the slope on the force experienced by the snowboarder.

(e) For snowboarders approaching the ledge with a non-zero velocity, the slope can be made so that the reaction force on landing is zero.

Discuss what shape of the slope would be required and why.

Assessor's use only

(8)

<table>
<tr><th>Question</th><th>Evidence</th><th>1-4 marks</th><th>5-6 marks</th><th>7-8 marks</th></tr>
<tr><td>THREE
(a)</td><td>GPE goes to KE – specify that the arm must be massless, otherwise there is rotational KE in the rotating beam.</td><td rowspan="4">Thorough understanding of this application of physics.

OR

Partially correct mathematical solution to the given problems

AND / OR

Partial understanding of this application of physics.</td><td rowspan="4">(Partially) correct mathematical solution to the given problems.

AND / OR

Reasonably thorough understanding of this application of physics.</td><td rowspan="4">Correct mathematical solution to the given problems.

AND

Thorough understanding of this application of physics.</td></tr>
<tr><td>(b)</td><td>$R = \frac{2v^2 \sin\alpha \cos\alpha}{g}$
v = velocity of projectile at release
α = angle of v to the horizontal (at release)

At max range $\alpha = 45^\circ$ so $\sin\alpha\cos\alpha = 0.5$
And $\frac{1}{2}mv^2 = \mathrm{M}gh$ (All of the GPE is converted to KE)

So $v^2 = \frac{2\mathrm{M}gh}{\mathrm{m}}$
and $R = \frac{2 \times 2\mathrm{M}gh \times 0.5}{\mathrm{mg}}$
$R = \frac{2\mathrm{M}h}{m}$</td></tr>
<tr><td>(c)</td><td>When the falling weight is dropped it swings BACKWARDS, so the trebuchet frame "wants" to go forward – centre of mass tries to stay in the same position; or conservation of momentum in a closed system. So yes, put it on wheels so it can roll forward and give the projectile some additional KE.</td></tr>
<tr><td>(d)</td><td>It would be the same (100 m).
• Equation for maximum range is independent of g.
• Force supplied by the falling counterweight is only 1/6 that supplied on the Earth but once launched the projectile is only subject to 1/6 of the force returning it to the ground. These two factors exactly cancel each other.</td></tr>
</table>

Question	Evidence	1-4 marks	5-6 marks	7-8 marks
SIX (a)	Gravitational energy transferred = $mgh + mgb$ Energy transferred to body (elastic, heat) = Fb (Average force × resulting movement of the C of M.) If these two are the same (ie if there is no "give" in the ground) then $Fb = mgh + mgb$ or $f = mg(1+\frac{h}{b})$ Can also be done by calculating the deceleration ($a = \frac{gh}{b}$) and summing forces in the vertical plane.	Partially correct mathematical solution to the given problems. AND / OR Partial discussion of the underlying physics of this application.	(Partially) correct mathematical solution to the given problems. AND Reasonably thorough discussion of the underlying physics of this application.	Thorough discussion of the underlying physics of this application. AND Correct mathematical solution to the given problems.
(b)	$Fd = \Delta E \qquad F = mg\frac{h+b}{b}$ $h = 3$ m , $b = 0.5$ m gives $F = 7mg$ And its only the average force - we are assuming constant acceleration for the duration of the stop and since the acceleration has to begin and end at zero and take some time to reach its maximum value, the maximum value must be larger than the $7mg$ calculated.			
(c)	$b = v_{av} \times t = \frac{vt}{2}$ $v = (2gh)^{0.5}$ (due to conservation of energy) $t = \frac{2b}{(2gh)^{0.5}} = b\left(\frac{2}{gh}\right)^{0.5}$ Effectively the distance b is increased as the snow sinks a bit on landing. Assuming b increases by 10 cm this will make $F = 6mg$. This is a significant reduction.			
(d)	The force normal to the surface will reduce as θ gets larger and $\cos\theta$ gets closer to zero. As the slope gets steeper the force normal to the surface will reduce.			
(e)	The answer wanted is a diagram showing the slope to be a parabola – the slope will match the freefall path of the snowboarder and so no force will be exerted at the time of (grazing) contact. However it should be noted that forces will have to be exerted at some time when the slope changes its profile (or else the projectile keeps going down forever).			