QUESTION TWO: BUNGY JUMPING

ASSESSOR'S USE ONLY

Acceleration due to gravity = 9.81 m s^{-2}

Standing on a platform that is 25.0 m above a river, Emma, of height 2.00 m and mass *m*, is tied to one end of an elastic rope (the bungy) by her ankles, while the other end of the bungy is fixed to a platform. The length of the bungy is adjusted so that Emma's downward motion stops at the instant her head reaches the water surface. When Emma is at rest, in equilibrium, at the end of the bungy, her head is 8.00 m above the water. The unstretched length of the bungy is *L*, and it has a spring constant of k. Assume Emma's centre of mass is halfway up her body.

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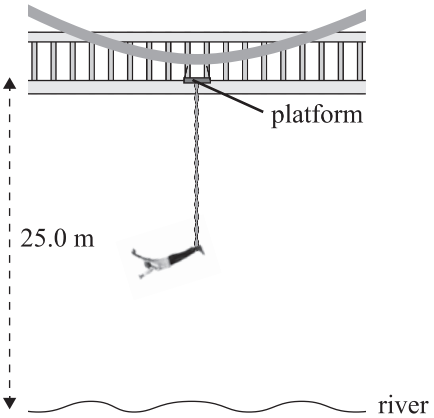
The bungy jump site

http://thebostonjam.files.wordpress.com/2011/11/heidi-jump1.jpg?w=474



(a) By considering energy conservation, show that at the lowest point in the jump, $mgh = \frac{1}{2}\text{k}(23 - L)^2$, where *h* is the change in height of Emma's centre of mass.

Explain all reasoning.

(b) Show that, at the equilibrium position, $mg = \text{k}(15 - L)$.

(c) Show that the value of L is 13.0 m.

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(d) (i) Calculate Emma's maximum speed.

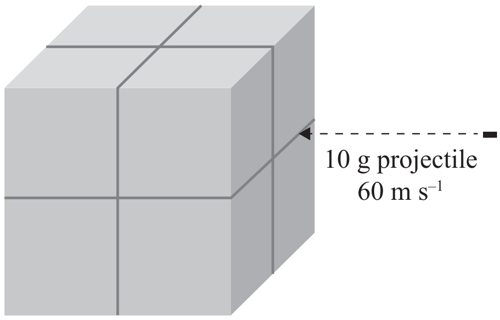
(ii) Calculate Emma's maximum acceleration.

(e) Explain what will happen to the spring constant of the bungy when its length is reduced by 50%.

(8)

QUESTION THREE: THE BLOCK

Eight small blocks, of dimensions 3 cm by 3 cm by 3 cm, are glued together to form a cube, as shown. Each block has a mass of 100 grams. The cube is placed on a frictionless surface and a 10 g projectile is fired into the cube at velocity of 60 m s^{-1}, as shown. The projectile enters the cube 3 cm from the base (through the horizontal plane of the centre of mass), and 1 cm from the right-hand edge. Assume that the projectile stops inside the cube on the same line as it entered.



(a) Show that once the projectile stops, the velocity of the centre of mass of the cube and projectile is 0.74 m s^{-1}.

(b) Show that once the projectile stops, the angular velocity of the cube and projectile is given by

$$\omega = \frac{m_{\text{projectile}}\, v_{\text{projectile}}\, d}{I + m_{\text{projectile}}\, r^2}$$

where d is the perpendicular distance from the centre of mass of the cube to the initial direction of the projectile, I is the rotational inertia of the cube (about a vertical axis through the centre of mass), r is the distance from the centre of mass to the final position of the projectile, and ω is the angular velocity of the cube and projectile.

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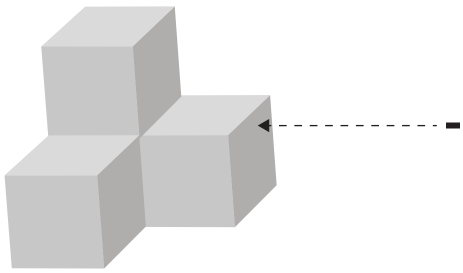
(c) Show that $\dfrac{m_{\text{projectile}}}{m_{\text{cube}} + m_{\text{projectile}}}$ is the fraction of the initial kinetic energy that goes into moving the centre of mass of the cube and projectile.

(d) Explain what has happened to the rest of the projectile's initial energy.

(e) Four blocks are removed from the cube to create a new object, as shown below.



If the projectile were fired at the new object (in the horizontal plane of the centre of mass of the new object and again 1 cm in from the right-hand edge of the object, as shown above), explain what differences would be seen in the motion of the new object compared to the original cube.

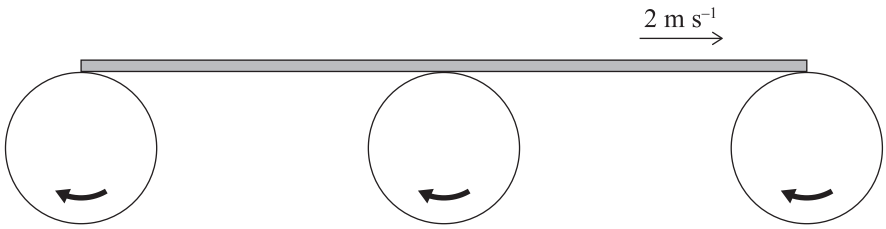
(8)

QUESTION FOUR: THE WOODEN SHEETS

Acceleration due to gravity = 9.81 m s^{-2}

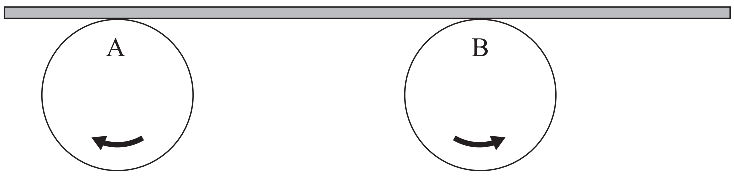
Uniform sheets of wood of mass 50 kg and length 2.40 m are moved around a production mill by powered rollers. After their initial acceleration, the wooden sheets move without slipping at a constant velocity of 2.00 m s^{-1}.

The frictional force between the rollers and the wood can be calculated using $F_{\text{friction}} = \mu N$, where μ is the coefficient of friction between the wood and the rollers, and N is the normal reaction force on the wood.



(a) Explain why no work is done on the wooden sheet when it is travelling at constant velocity.

(b)



One of the rollers has a drive gear malfunction and rotates in the opposite direction, as shown above. The wooden sheet is initially displaced towards the right-hand roller.

Show, by taking moments about the centre of mass of the sheet, that the normal reaction force acting through point A is $N_{\text{A}} = \dfrac{mg(d-x)}{2d}$ where $2d$ is the distance between the centres of the two rollers, and x is the displacement of the centre of mass of the wooden sheet from the midpoint between the rollers.

ASSESSOR'S USE ONLY

(c) It can be shown that the normal reaction force at point B is $N_B = \dfrac{mg(d+x)}{2d}$.

Using this result and the result from (b) above, show that the sheet undergoes simple harmonic motion described by the following expression $F = \dfrac{-\mu mgx}{d}$.

(d) (i) Show that the period of the oscillation is 2.46 s when d is 0.600 m and μ is 0.400.

(ii) An object is dropped onto the oscillating sheet of wood.

If the coefficient of friction between the object and the wood is 0.200, show that the maximum amplitude with which the wood can move horizontally without causing the object to slip is 0.300 m.

(8)

Q	Evidence	1–4 marks	5–6 marks	7–8 marks
TWO (a)	At the top of the jump, Emma's centre of mass is 1 m above the platform. At the bottom of the jump, Emma's centre of mass is 1 m above the river. So Emma's centre of mass moves 25 m, from the platform to the bottom of the jump. Loss of gravitational potential energy is mgh. Extension of bungy spring is $x = (25 - 2) - L$. Gain in potential energy of the spring is $0.5kx^2 = 0.5k(23 - L)^2$ Therefore, gravitational potential energy = spring potential energy so $mgh = 0.5\text{k}(23 - L)^2$	Thorough understanding of these applications of physics. OR Partially correct mathematical solution to the given problems. AND / OR Partial understanding of these applications of physics.	(Partially) correct mathematical solution to the given problems. AND / OR Reasonably thorough understanding of these applications of physics.	Correct mathematical solution to the given problems. AND Thorough understanding of these applications of physics.
(b)	At equilibrium, downward force of gravity = upward force of spring $m\text{g} = \text{k}x = \text{k}(25 - 10 - L)$ ie, $m\text{g} = \text{k}(15 - L)$			
(c)	From Two (a) and Two (b), equating expressions for mgh $0.5\text{k}(23 - L)^2 = \text{k}(15 - L)h$ The "k"s cancel, and h is 25 m. So $0.5\,(23 - L)^2 = (15 - L)25$ therefore $(23 - L)^2 = (15 - L)50$ $529 + L^2 - 46L = 750 - 50L$ $L^2 + 4L - 221 = 0$ Solving this and taking the positive root gives $L = 13$ m.			
(d)(i)	Maximum speed occurs at the point that Emma feels zero acceleration (before this the acceleration is downwards, and after this the acceleration is upwards, which slows the velocity). Zero acceleration when $m\text{g} = \text{k}x$ We know that $\dfrac{m\text{g}}{\text{k}} = 2$. Therefore, zero acceleration at $x = 2$. Loss of potential energy at $x = 2$ is $m\text{g}(L + 2 + 2)$ $= m\text{g}(L + 4)$ Gain of spring potential energy at $x = 2$ is $0.5\text{k}2^2 = 2\text{k}$ Therefore, $\text{KE} = m\text{g}(13 + 4) - 2\text{k} = 17\,m\text{g} - 2\text{k}$ $0.5\,mv^2 = 17\,m\text{g} - 2\text{k}$ $v^2 = 34\text{g} - 4\dfrac{\text{k}}{m} = 34\text{g} - 2\text{g}$ (from Two (b)) $= 32\text{g} = 32(9.81)$ $= 313.92$ therefore, $v = 17.7\text{ m s}^{-1}$			
(ii)	Maximum acceleration downwards is g. But we know that the bungy counteracts this and actually turns around the motion. Maximum acceleration due to bungy is $\dfrac{\text{k}x}{m}$ and is maximum when x is a maximum. Maximum value of x is 10 m, and $\dfrac{\text{k}}{m} = \dfrac{g}{2}$ (from Two (b)). So maximum acceleration due to bungy is 5g (49.05 m s^{-2}). Subtract the acceleration due to gravity, to get maximum of 4 g (39.24 m s^{-2}).			
(e)	If force F is applied to the bungy, the change in length of the whole bungy will be x. If we imagine the bungy to be made of two parts, each part will extend by $\dfrac{x}{2}$. Since the force is the same, the spring constant has doubled.			

Q	Evidence	1–4 marks	5–6 marks	7–8 marks
THREE (a)	Momentum of projectile = $0.01 \text{ kg} \times 60 \text{ m s}^{-1}$ Momentum of cube plus projectile = $0.81 \text{ kg} \times v \text{ m s}^{-1}$ Equating them, $0.01 \times 60 = 0.81v$ $v = \dfrac{0.6}{0.81} = 0.741 \text{ m s}^{-1}$ ie $v = \dfrac{m_{\text{projectile}} V_{\text{projectile}}}{m_{\text{projectile}} + M_{\text{cube}}}$	Thorough understanding of these applications of physics. OR Partially correct mathematical solution to the given problems. AND / OR Partial understanding of these applications of physics.	(Partially) correct mathematical solution to the given problems. AND / OR Reasonably thorough understanding of these applications of physics.	Correct mathematical solution to the given problems. AND Thorough understanding of these applications of physics.
(b)	Angular momentum initially = mvr $= m_{\text{projectile}} V_{\text{projectile}}\, d$ This is the angular momentum of the cube plus projectile = $I\omega + m_{\text{projectile}} r^2 \omega$ $I\omega + m_{\text{projectile}} r^2 \omega = m_{\text{projectile}} V_{\text{projectile}} d$ $\omega = \dfrac{m_{\text{projectile}} V_{\text{projectile}} d}{I + m_{\text{projectile}} r^2}$			
(c)	Initial $KE = 0.5 m_{\text{projectile}} V_{\text{projectile}}^2$ Final $KE = \dfrac{0.5\left(m_{\text{projectile}} + M_{\text{cube}}\right)\left(m_{\text{projectile}} V_{\text{projectile}}\right)^2}{\left(m_{\text{projectile}} + M_{\text{cube}}\right)^2}$ $\dfrac{\text{Final } KE}{\text{Initial } KE} = \dfrac{0.5\left(m_{\text{projectile}} + M_{\text{cube}}\right)\left(m_{\text{projectile}} V_{\text{projectile}}\right)^2}{\left(m_{\text{projectile}} + M_{\text{cube}}\right)^2} \Big/ 0.5 m_{\text{projectile}} V_{\text{projectile}}^2$ $= \dfrac{m_{\text{projectile}}}{\left(m_{\text{projectile}} + M_{\text{cube}}\right)}$			
(d)	Some energy has gone to rotational KE, $0.5I'\omega^2$, where I' = moment of inertia of cube plus projectile. The remainder has been lost as heat and in deforming the blocks.			
(e)	The shape will have its centre of mass move with approximately twice the velocity of the original cube (because the mass is half as much). The moment of inertia is significantly reduced and so the final angular velocity must also increase (L is slightly reduced but I is reduced significantly leading to an increase in angular velocity).			

<table>
<tr><th>Question</th><th>Evidence</th><th>1–4 marks</th><th>5–6 marks</th><th>7–8 marks</th></tr>
<tr><td>FOUR
(a)</td><td>The work-energy theorem states that work done = ΔE_K.
Since no change has occurred to the velocity and therefore the kinetic energy, no work has been done on the sheets.</td>
<td rowspan="6">Thorough understanding of these applications of physics.

OR

Partially correct mathematical solution to the given problems.

AND / OR

Partial understanding of these applications of physics.</td>
<td rowspan="6">(Partially) correct mathematical solution to the given problems.

AND / OR

Reasonably thorough understanding of these applications of physics.</td>
<td rowspan="6">Correct mathematical solution to the given problems.

AND

Thorough understanding of these applications of physics.</td></tr>
<tr><td>(b)</td><td>$N_A(d + x) = N_B(d - x) = (mg - N_A)(d - x)$
Since $mg = N_A + N_B$

Therefore, $N_A(d + x + d - x) = mg(d - x)$
$N_A = \dfrac{mg(d-x)}{2d}$</td></tr>
<tr><td>(c)</td><td>$F_A = \dfrac{\mu mg(d-x)}{2d}$
$F_B = \dfrac{\mu mg(d+x)}{2d}$
These two frictional forces are oppositely directed, and so
$F_{net} = \dfrac{\mu mg 2x}{2d} = \dfrac{\mu mgx}{d}$

However, if the sheet moves to the right (positive x), roller B will move it to the left, and vice versa for A.
So $F_{net} = -\dfrac{\mu mgx}{d}$
This is simple harmonic motion as the force is proportional to the displacement but oppositely directed.</td></tr>
<tr><td>(d)(i)</td><td>$\omega^2 = \dfrac{\mu g}{d}$ (from the SHM equation above)
$\omega = \dfrac{2\pi}{T}$
$T^2 = \dfrac{d 4\pi^2}{\mu g}$
Substituting shows that $T = 2.46$ s</td></tr>
<tr><td>(ii)</td><td>Friction = $\mu N = 0.2N = 0.2Mg$ (M = mass of the object)
Max acceleration without slipping is $0.2g$
Max acceleration = $A\omega^2$
$A = \dfrac{0.2g}{\omega^2}$ = 0.300 m (since $\omega^2 = \dfrac{\mu g}{d}$)</td></tr>
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