## QUESTION FOUR: THE SLIDING TOY (8 marks)

Acceleration due to gravity $=9.80 \mathrm{~m} \mathrm{~s}^{-2}$
Benjamin is playing with his toy slider and track. The track has a circular loop of diameter, $D$, as shown in the diagram. Friction between the slider and the track is negligible.

(a) Explain the physical conditions under which the slider can travel around the loop without losing contact with the track.
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(b) If Benjamin releases the slider from rest, derive an expression for the minimum height, $h$, so that the slider does not lose contact with the track.
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(c) Explain how the minimum height changes for the following three different cases:
(i) Benjamin pushes the slider as he releases it.
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(ii) Benjamin replaces the slider with one that has greater mass, but is otherwise identical.
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(iii) Benjamin replaces the loop with an elliptical one with the same height as the circular one, as shown in the diagram.


## QUESTION FIVE: THE ASTRONAUT (8 marks)

Nicole, an astronaut, is assigned to make external repairs to a second space vehicle which is travelling in a direction parallel to her own, but with a greater relative velocity of $2.6 \mathrm{~m} \mathrm{~s}^{-1}$. The second space vehicle is 85 m in length. Nicole launches herself directly towards the nose of the vehicle with a speed of $6.0 \mathrm{~m} \mathrm{~s}^{-1}$ in a direction perpendicular to the motion of the second space vehicle. At this point, the vehicles are separated by a distance of 200 m . As a safety measure, Nicole is equipped with a launcher that fires a magnetic puck. The magnetic puck is attached to a 4 m length of rope, the other end of which is tied to Nicole. The magnetic puck can only be fired in the direction in which Nicole is travelling. The magnetic puck has a mass equal to $1 \%$ of Nicole's mass, but only $0.004 \%$ of the mass of the second space vehicle that she is heading towards.

(a) Show that the magnetic puck allows Nicole to attach herself to the second space vehicle.
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(b) Nicole can control the speed at which she fires the puck, from $10 \mathrm{~m} \mathrm{~s}^{-1}$ to $750 \mathrm{~m} \mathrm{~s}^{-1}$ relative to herself.

If Nicole fires the magnetic puck when she is 4 m away from the second space vehicle:
(i) Discuss the effect of the puck velocity on the success of the mission.
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(ii) Discuss the effect of the puck velocity on the second space vehicle, for the full range of puck velocities.
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| Question | Evidence | 1-4 marks | 5-6 marks | 7-8 marks |
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| 4(a) | In order to go round the loop, the centripetal force at the top has to be provided by the weight + reaction force. (In the minimum case the reaction could be zero). <br> Mathematically $\frac{m v^{2}}{r} \geq m g$ | Partially correct mathematical solution to the given problems <br> AND/OR | (Partially) <br> Correct mathematical solution to the given problems <br> AND | Thorough discussion of the underlying physics of this application <br> AND |
| (b) | Initial energy $=m g h$ <br> At the top of the loop energy = kinetic energy+potential energy $=\frac{1}{2} m v^{2}+m g D$ <br> At top minimum speed $\begin{aligned} & \frac{m v^{2}}{r}=m g \quad\left(r=\frac{D}{2}\right) \\ & \Rightarrow v^{2}=\frac{D g}{2} \end{aligned}$ <br> So combining $\begin{aligned} & m g h=\frac{1}{2} m \frac{D g}{2}+m g D \\ & h=\frac{1}{4} D+D \\ & h=\frac{5}{4} D \end{aligned}$ | Incomplete discussion of the underlying physics of this application <br> OR <br> Correct mathematical solution to the given problems <br> OR <br> Thorough discussion of the underlying physics of this application. | Reasonably thorough discussion of the underlying physics of this application. | Correct mathematical solution to the given problems. |
| (c)(i) | Pushing the slider means it has more kinetic energy and so requires less potential energy to reach the energy total needed to complete the loop. Therefore less height is needed. |  |  |  |
| (ii) | The height is independent of mass (assuming no friction) so no difference. |  |  |  |
| (iii) | In the ellipse the radius of curvature at the top is smaller (less than $D / 2$ ). <br> If the radius is less then less speed is required at the top ( $v^{2}=r g$ ). <br> $D$ is the same so potential energy at top is the same but less kinetic energy is required at the top so the total energy is less so the minimum height is lower. |  |  |  |


| Question | Evidence | 1-4 marks | 5-6 marks | 7-8 marks |
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| 5(a) | When the puck is 4 m in front of Nicole the centre of mass is $4 / 101 \mathrm{~m}$ in front of her and the puck is 3.9604 m in front of the CoM. <br> Space Vehicle 2 takes 32.69 s to go 85 m ( 85 / 2.6). The CoM of Nicole and the puck will travel 196.154 m in this time ( $32.69 \times 6$ ). <br> With the puck being 3.9604 m in front of the CoM, 200.114 m is the greatest reach of the puck and enough to reach the craft in time. | Some understanding of at least two important aspects of the physics of the situation outlined. | Clear understanding of two important aspects of the physics of the situation outlined. | Clear understanding of all important aspects of the physics of the situation outlined. |
| (b)(i) | From above: <br> At $6.0 \mathrm{~m} \mathrm{~s}^{-1}$ it takes $200 / 6 \mathrm{~s}$ for Nicole to reach Space Vehicle $2(\mathrm{SV} 2)=33.33$ seconds. <br> Nicole only has the time that SP2 has to move 85 m to make contact - that time is $85 / 2.6 \mathrm{~s}=32.69 \mathrm{~s}$. <br> It takes $196 / 6 \mathrm{~s}(32.67 \mathrm{~s})$ to get within 4 m of SP2. <br> In 32.69 s Nicole only travels $196.154 \mathrm{~m}(32.69 \times 6)$. <br> Puck has $32.69-32.67$ seconds ( 0.02564 s ) to move the 4 m . <br> This is equivalent to an average velocity of $156 \mathrm{~m} \mathrm{~s}^{-1}$. <br> Any speed lower than this will result in failure of the mission. <br> Using conservation of momentum: <br> When Nicole fires the puck forwards she will recoil backwards (the CoM will continue at $6 \mathrm{~m} \mathrm{~s}^{-1}$ forwards regardless). <br> Looking at the extreme case: <br> Results in a velocity of Nicole of $7.44 \mathrm{~m} \mathrm{~s}^{-1}$ backwards since she was already going at $6 \mathrm{~m} \mathrm{~s}^{-1}$ forwards she will now go $1.44 \mathrm{~m} \mathrm{~s}^{-1}$ backwards (from conservation of momentum). At this speed it is still possible for a successful mission. As long as the puck and Nicole come to a dead stop at the end of the explosion - the CoM will 0.15384 m further on from the 4 m point at the end of the explosion. This will mean that if the puck can travel greater than 3.84616 m during the explosion then the mission will be a success. The time of the explosion will be $4 /\left(v_{\mathrm{p}}+v_{\mathrm{n}}\right)$ so the distance travelled by the puck in relation to the CoM will be $4 \mathrm{v}_{\mathrm{p}} /\left(v_{\mathrm{p}}+v_{\mathrm{n}}\right)$ if this is greater than 3.84616 m then the puck will make it. This will always be the case for velocities greater than about $150 \mathrm{~m} \mathrm{~s}^{-1}$. Consideration will also be given to answers that show particular physical insight - such as discussion involving the enormous accelerations that would be experienced. The fact that the above model assumes that the puck and Nicole do not recoil when they reach the 4 m extension. The fact that energy losses will result in the explosions. |  |  |  |
| (ii) | At low velocity the puck does not collide so has no effect. At a speed of $750 \mathrm{~m} \mathrm{~s}^{-1}$ by considering conservation of momentum it can be shown that $4 \times 10^{-5} \times \mathrm{M} \times 750$ (downwards) $+\mathrm{M} \times 2.6$ (across) $=\mathrm{Mv}_{\text {new }}$ The acquired momentum will have little effect. ( 0.03 compared to 2.6). <br> It will result in a torque on SP2. <br> In terms of energy at say $300 \mathrm{~m} \mathrm{~s}^{-1}$ the energy of impact is similar to the kinetic energy of SP2 - there will be considerable damage caused. At higher velocities this effect will obviously be greater. |  |  |  |

