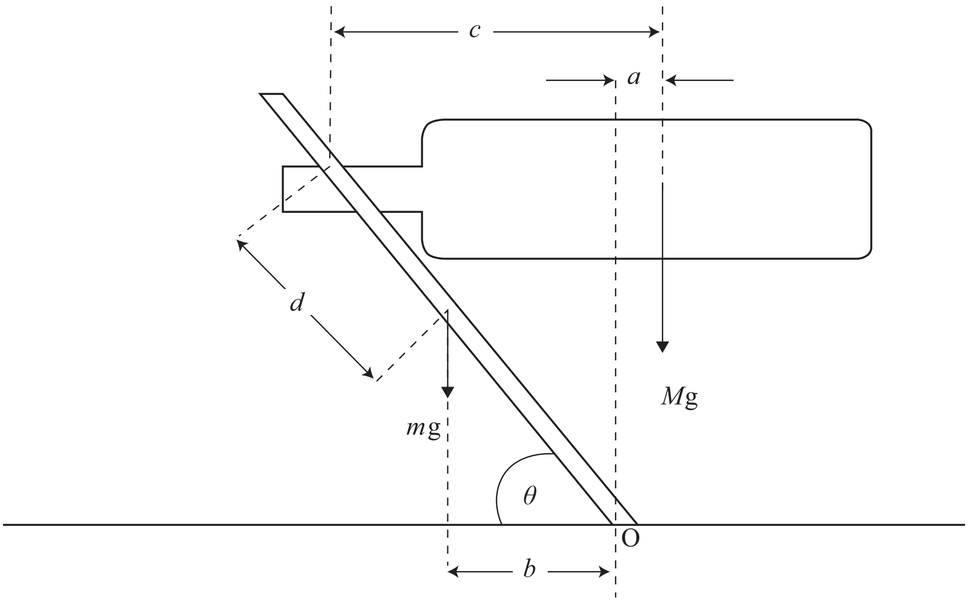
## QUESTION TWO: THE WINE BUTLER (8 marks)

Assessor's use only

The "Wine Butler" is a simple device, usually made of wood or plastic, that is used to hold a bottle of wine in an attractive and interesting manner. The neck of the wine bottle is placed through a hole in the wine butler, and the butler is set at an angle so that the whole assembly is balanced, as shown.

The wine butler has a length of $2L$ and it can be assumed that its centre of mass is in the middle, L from either end. In the diagram, the distances from the top of the hole to the centres of mass of the bottle and the butler are indicated, along with the horizontal distances from the respective centres of mass to the point of balance, O.




(a) (i) By taking torques, show that the wine butler is in rotational equilibrium if:

$$\frac{b}{a} = \frac{M}{m}$$

Assessor's use only

(ii) Show that $\cos\theta = \dfrac{Mc}{(m+M)L + Md}$

(b) Under what conditions will it **not** be possible to make the system stable?

(c) If $L$ is increased, what will happen to the angle? Explain, using physical arguments.

Assessor's use only

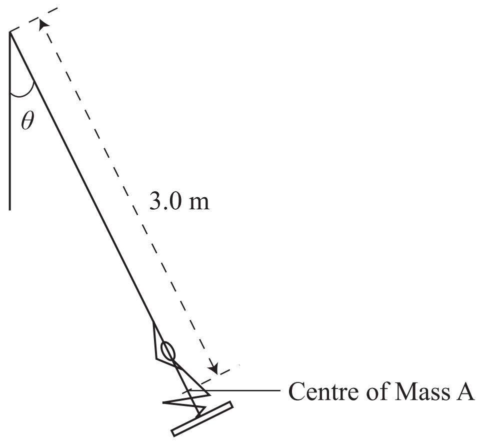
(d) After the contents of the bottle have been removed, can the bottle be replaced in the same equilibrium position or must the neck of the bottle be moved to the left or right? (Assume that the centre of mass of the bottle stays in the same place when the wine is removed.)

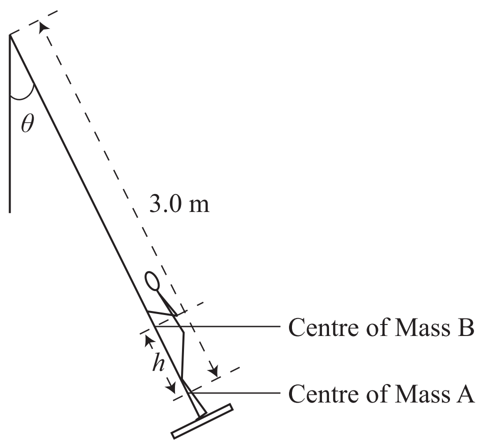
(8)

**QUESTION THREE: THE PHYSICS OF THE SWING** (8 marks)

Assessor's use only

Acceleration due to gravity = 9.80 m $s^{-2}$




Kristina is playing on a swing. She knows that she can alter the amplitude of the swing by changing her position at various stages of the swing cycle. Initially, Kristina is squatting on the swing, with her feet on the seat, as shown in the top figure.

(a) When the swing is at angle $\theta$, Kristina stands up so that her centre of mass changes by a distance $h$ (from A to B), as shown in the bottom figure.

Show that Kristina gains gravitational potential energy of $mgh\cos\theta$.

Assessor's use only

(b) When Kristina stands she notices that the swing picks up speed.

(i) Explain why the speed increases.

(ii) Explain whereabouts in the motion Kristina should stand up in order to gain the maximum kinetic energy.

(c) By considering the energy changes at the lowest and highest points of Kristina's motion, show that if Kristina stands up at the lowest point of the swing and squats down at the highest point of the swing, the increase in mechanical energy is given by:

$$\Delta E = \left(\frac{I_1^{\,2} - I_1 I_2}{2I_2}\right)\omega_1^{\,2} + mgh(1 - \cos\alpha)$$

where

$I_1$ = moment of inertia of the swing and Kristina while squatting
$I_2$ = moment of inertia of the swing and Kristina while standing
$\omega_1$ = angular velocity as the swing reaches its lowest point
$\alpha$ = angle reached at the end of the swing

Assessor's use only

(d) Kristina and the swing have a combined mass of 50 kg. When she squats, the centre of mass is 3 m from the point of rotation. When she stands up, the centre of mass is 2.6 m from the point of rotation.

With Kristina squatting on the swing, she is released from rest at an angle of 30° ($\theta = 30°$). As she passes through the lowest part of the motion, she quickly stands up.

What angle does the swing reach on the far side?
(Kristina plus the swing seat can be modelled as a single point mass on the end of a simple pendulum.)

Assessor's use only

(8)

| Question | Evidence | 1-4 marks | 5-6 marks | 7-8 marks |
|---|---|---|---|---|
| 2(a)(i) | If the system is stable the sum of turning forces on the system will be zero.<br>Taking moments about O, there are only two torques, *M* g *a* (clockwise) and *m* g *b* ( anticlockwise). These two must be equal if the system is to be stable.<br>Taking external torques around the point of balance,<br>*Mga* – *mgb* = 0 => *Ma* = *m*b => *M*/*m* = *b*/*a* | Partially correct mathematical solution to the given problems.<br><br>AND / OR<br><br>Partial understanding of this application of physics. | (Partially) correct mathematical solution to the given problems.<br><br>AND / OR<br><br>Reasonably thorough understanding of this application of physics. | Correct mathematical solution to the given problems.<br><br>AND<br><br>Thorough understanding of this application of physics. |
| (ii) | $\cos\theta = \frac{b}{L}$ and $\cos\theta = \frac{(c-a)}{(L+d)}$<br>Therefore, $\frac{b}{L} = \frac{(c-a)}{(L+d)}$<br>$\Rightarrow \frac{\left(\frac{aM}{m}\right)}{L} = \frac{(c-a)}{(L+d)}$ $\left[\text{from } \frac{M}{m} = \frac{b}{a}\right]$<br>$\Rightarrow \frac{a(L+d)M}{mL} = c-a$<br>$\Rightarrow c = \frac{a(mL+(L+d)M)}{mL}$<br>$\Rightarrow c = \left(\frac{\cos\theta}{M}\right)(mL+(L+d)M)$ $\left[\text{from } \cos\theta = \frac{b}{L} = \frac{aM}{mL}\right]$<br>$\Rightarrow \cos\theta = \frac{Mc}{(mL+(L+d)M)}$<br>$\Rightarrow \cos\theta = \frac{Mc}{(m+M)L+Md}$ | | | |
| (b) | If *M* is too large and / or *c* is too big, then $\cos\theta$ will be greater than 1. This cannot happen – the system will rotate so that the bottle hits the bench. | | | |
| (c) | $\cos\theta$ will drop as *L* is in the denominator and therefore $\theta$ will increase.<br><br>Assuming constant mass as L increases, b increases so the anticlockwise torque increases. This is unstable; to return to equilibrium conditions $\theta$ must increase. | | | |
| (d) | When the wine is removed, *M* will decrease, so the clockwise torque will decrease. Moving the neck of the bottle to the right will increase the torque again by increasing the distance from the centre of mass over which the weight force acts. | | | |

| Question | Evidence | 1-4 marks | 5-6 marks | 7-8 marks |
|---|---|---|---|---|
| 3(a) | Change in height = AB $\cos\theta = h\cos\theta$<br>GPE = $mg$ × change in height = $mgh\cos\theta$ | Thorough understanding of this application of physics.<br><br>OR<br><br>Partially correct mathematical solution to the given problems.<br><br>AND / OR<br><br>Partial understanding of this application of physics. | (Partially) correct mathematical solution to the given problems.<br><br>AND / OR<br><br>Reasonably thorough understanding of this application of physics. | Correct mathematical solution to the given problems.<br><br>AND<br><br>Thorough understanding of this application of physics. |
| (b)(i) | Her angular momentum ($I\omega$) is unchanged because no external torques act but her moment of inertia ( $I \propto mr^2$ ) decreases because her mass moves nearer the centre of the angular motion. If $I$ decreases, the angular velocity $\omega$ must increase to keep the angular momentum constant | | | |
| (b)(ii) | $\Delta E_K = E_2 - E_1 = \frac{1}{2} I_2\omega_2^2 - \frac{1}{2} I_1\omega_1^2$<br>(from b) $\omega_2 = \frac{I_1\omega_1}{I_2}$<br>so $\Delta E_K = \frac{1}{2}\omega_1^2\left(\frac{I_1^2}{I_2} - I_1\right)$<br>$\Delta E_K = \left(\frac{I_1^2 - I_1 I_2}{2I_2}\right)\omega_1^2$<br>Maximum gain in KE is at the bottom of the swing, when $\omega_1$ is at its maximum value. | | | |
| (c) | The increase in gravitational potential energy at the bottom of the swing = $mgh$<br>At the bottom of the swing, the increase in rotational kinetic energy = $\frac{1}{2} I_2\omega_2^2 - \frac{1}{2} I_1\omega_1^2$<br>But $I_1\omega_1 = I_2\omega_2 \Rightarrow \omega_2 = \frac{I_1\omega_1}{I_2}$<br>Therefore, incr. in rot. energy $= \frac{1}{2} I_2\omega_2^2 - \frac{1}{2} I_1\omega_1^2$<br>$= \frac{1}{2} I_2\left(\frac{I_1\omega_1}{I_2}\right)^2 - \frac{1}{2} I_1\omega_1^2 = \frac{1}{2} I_1\omega_1^2\left(\frac{I_1}{I_2} - 1\right)$<br>$= \frac{I_1^2\omega_1^2 - I_1 I_2\omega_1^2}{2I_2}$<br>The total change in energy at the bottom is the change in rotational energy plus change in potential energy<br>$= \frac{I_1^2\omega_1^2 - I_1 I_2\omega_1^2}{2I_2} + mgh$<br>And the change in energy at the top is clearly $-mgh\cos\alpha$<br>Therefore net change of energy<br>$= \left(\frac{I_1^2 - I_1 I_2}{2I_2}\right)\omega_1^2 + mgh - mgh\cos\alpha$ | | | |

| Question | Evidence | 1-4 marks | 5-6 marks | 7-8 marks |
|---|---|---|---|---|
| (d) | GPE lost = Rotational KE gained<br>$mgh = \frac{1}{2} I_1 \omega_1^2$<br>$h = 3 - 3\cos\theta \quad I_1 = mr^2 \ (r = 3\,\text{m})$<br>$so\ \omega_1^2 = \frac{(2 \times 9.80 \times (3 - 3\cos\theta))}{9}$<br>After the stand up<br>$\omega_2 = \frac{I_1 \omega_1}{I_2}$<br>$mgh_{\text{new}} = \frac{1}{2} I_2 \omega_2^2$<br>$50 \times 9.80 \times h_{\text{new}} = \frac{1}{2} \frac{(50 \times 3^2)^2 \omega_1^2}{50 \times 2.6^2}$<br>$h_{\text{new}} = \frac{9(3 - 3\cos 30°)}{2.6^2} = 0.535106\ \text{m}$<br>$\cos\theta_{\text{new}} = \frac{2.6 - 0.535106}{2.6}$<br>$\theta_{\text{new}} = 37.4°$ | Partially correct mathematical analysis of given problems<br><br>And/or<br><br>Partial discussion of the underlying physics of the LCR resonant circuit<br><br>OR<br><br>Correct mathematical analysis of given problems | (Partially) correct mathematical analysis of given problems<br><br>And<br><br>Reasonably thorough discussion of the underlying physics of the LCR resonant circuit | Correct mathematical analysis of given problems<br><br>And<br><br>Thorough discussion of the underlying physics of the LCR resonant circuit |