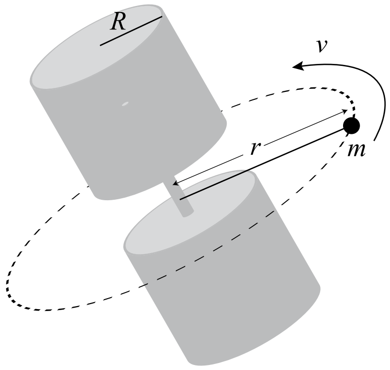
QUESTION THREE (6 marks)

Assessor's use only

A large satellite in orbit has a mechanism that allows a camera to revolve around it. The camera has mass m and is travelling at speed v on the end of a cord of length r which extends to the centre of the satellite. The satellite is cylindrical in shape with radius R.



The initial speed of the camera is v_0 and the initial radius of its circular path is r_0. A mechanism within the satellite allows the cord to be drawn in so that the radius of revolution for the camera decreases.

(a) Suppose that the cord begins to be drawn in towards the satellite at a constant rate. Express the speed v of the camera in terms of r, r_0 and v_0. Ignore friction.

(b) In order to bring the camera within the radius of the satellite (ie $r \leq R$), the camera must be drawn inwards by the cord. Derive an expression for the tension as the radius of the camera's orbit is reduced, in terms of m, r, r_0 and v_0. Explain what will happen to the tension as the camera is brought towards the centre of the satellite.

(c) Derive an expression for the amount of work required to bring the camera from radius r_0 to radius R.

Assessor's use only

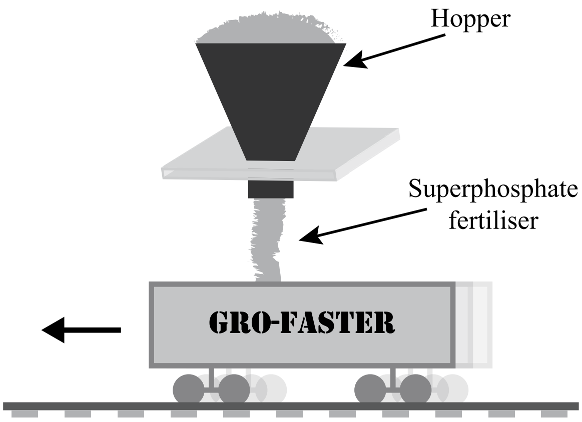
(6)

Assessor's use only

SECTION B

QUESTION ONE: "GRO-FASTER" FERTILISER FACTORY (8 marks)

At the "Gro-Faster" fertiliser factory, superphosphate fertiliser is transferred from a hopper into railway wagons, which are directly under the hopper as the superphosphate is released (see diagram below). An empty railway wagon has a mass of 2.20×10^4 kg and each wagon has a speed of $1.25\ \text{m s}^{-1}$ as it approaches the hopper. Wagons are not connected with each other.



1.5×10^4 kg of superphosphate fertiliser are transferred from the hopper to each wagon.

(a) Calculate the momentum and velocity of a wagon after the superphosphate has been transferred, ignoring friction.

(b) One wagon has a hole in its floor, which allows some of the superphosphate to fall below the wagon as it rolls along the track. Discuss the effect, if any, this will have on the motion of the wagon, ignoring friction.

Assessor's use only

(c) The density of superphosphate fertiliser is 1.1×10^3 kg m^{-3} and the wagons are 1.5 m wide and 1.5 m high. Estimate the maximum mass of superphosphate that can be transferred in one hour.

(d) In fact, there is some friction, so to keep the wagons rolling at a constant speed, the track slopes downward at an angle θ. The frictional force is given by $F = \mu N$, where μ is the coefficient of friction between the rotating wheel and the track and N is the normal force of the track on the wheel. If the coefficient of friction is 0.005, calculate the angle of the track so that the wagons maintain a constant speed.

(8)

Question	Typical evidence that will be awarded one mark (if applicable)	Typical evidence that will be awarded two marks	
3(a)		No external torque so angular momentum is conserved (mass of camera small so position of centre of mass effectively centre of satellite). $L = mvr$ initially $L_0 = mv_0r_0$ as L conserved $mvr = mv_0r_0$ $v = \frac{v_0r_0}{r}$	2
3(b)	Only ONE aspect covered.	BOTH aspects covered as below. **Aspect 1:** The tension provides the centripetal force $T = F_c = \frac{mv^2}{r}$ from before $v = \frac{v_0r_0}{r}$ $\Rightarrow T = \frac{mv_0^2r_0^2}{r^3}$ **Aspect 2 :** as $r \rightarrow 0$ the tension $\rightarrow \infty$. This is a $1/r^3$ relationship so the force required will increase rapidly as the camera moves towards the centre.	2
3(c)		$\Delta W = \Delta E$ Work done $= \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2$ $= \frac{m}{2}\left(\frac{v_0^2r_0^2}{R^2} - v_0^2\right)$ $= \frac{mv_0^2}{2R^2}\left(r_0^2 - R^2\right)$ **OR** integration can be used to solve this problem.	2

SECTION B:

Question	Typical evidence that will be awarded one mark (if applicable)	Typical evidence that will be awarded two marks	
1(a)		Initial momentum $= 2.20 \times 10^4 \times 1.25$ $= 2.75 \times 10^4 \,\text{kg m s}^{-1}$ no external forces act so momentum conserved. $\therefore$ final momentum of wagon $= 2.75 \times 10^4 \,\text{kg m s}^{-1}$ velocity of wagon $= \frac{p}{m} = \frac{2.75 \times 10^4 \,\text{kg m s}^{-1}}{3.7 \times 10^4 \,\text{kg}}$ $= 0.74 \,\text{m s}^{-1}$	2
1(b)		If zero friction then momentum conserved. As the superphosphate falls out the mass of the wagon decreases. However, the velocity of the wagon doesn't increase as the falling phosphate has a horizontal velocity and therefore momentum. It is only when the phosphate hits the ground that it loses its momentum i.e. the wagon is losing mass and losing momentum but maintaining a constant velocity. (Alternative approach is to consider the net force in the *x* direction, which is zero and therefore the acceleration is zero.)	2
1(c)	Award 1 mark if used faster speed. Final answer will be closer to 1.1×10^7 kg.	volume of wagon $= 1.5 \times 1.5 \times$ length mass per wagon (assume full wagon) $= 1.5 \times 10^4 = \rho V = 1.1 \times 10^3 \times 1.5 \times 1.5 \times$ length $\therefore$ length $= 6.06 \,\text{m}$ If moving at $0.74 \,\text{m s}^{-1}$ time to fill 1 wagon $= \frac{d}{v} = \frac{6.06}{0.74} = 8.2 \,\text{s}$ in one hour can fill n wagons $n = \frac{60 \times 60}{8.2} = 440$ wagons therefore mass per hour $= 440 \times 1.5 \times 10^4$ $= 6.6 \times 10^6 \,\text{kg}$	2